

# The magnetic penetration depth influenced by the proximity to the surface

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The effect of smooth inhomogeneities near a superconductor boundary on the magnetic penetration depth  $\lambda$  is studied with emphasis on the proximity-induced spatial dependence of the Cooper pair amplitude. The influence of surface pair breaking or pair formation on  $\lambda$  is described within the Ginzburg-Landau theory, with no model assumptions, for both strongly type-II and strongly type-I homogeneous superconductors. Generic values of  $\lambda$ , which can differ greatly from the London penetration depth, are identified and demonstrated to be induced by large-scale inhomogeneities, when superconductivity is strongly suppressed on the surface.

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## I. INTRODUCTION

The magnetic penetration depth  $\lambda$  is the fundamental superconductor length scale related to the Meissner effect.<sup>1</sup> Measurements of temperature dependent  $\lambda$  provide valuable information on the superfluid density and the momentum-space structure of the gap function. They had an important role in identifying the nodal lines of the d-wave order parameter in cuprates<sup>2,3</sup> and in providing several important insights into the behavior of Fe-based superconductors.<sup>4</sup> For a homogeneous superconducting state, the corresponding data represent the bulk characteristics. The situation, in general, is different in the presence of inhomogeneities.

Changes of  $\lambda$  can be induced by spatially dependent material composition or structure, or by disorder and other defects. They have been considered theoretically for a long time, in particular within the London and the Ginzburg-Landau (GL) theories with spatially dependent coefficients of the bulk free energy.<sup>5-10</sup> An inhomogeneous state in superconductors can also arise due to pair breaking, or pair forming sample surfaces and/or interfaces. The surface/interface pair breaking can be induced by proximity to superconductor-normal metal interfaces and to magnetically active boundaries in various superconductors, including isotropic s-wave ones.<sup>11-15</sup> In unconventional superconductors the surface pair breaking can be present also near superconductor-insulator and superconductor-vacuum boundaries.<sup>16-23</sup> In a number of cases superconductivity can be locally enhanced<sup>24-29</sup>, for example, near a mutual interface with an adjacent superconductor possessing a higher critical temperature.

An inherent feature of the surface pair breaking is a spatial dependence of the pair amplitude over the coherence length  $\xi$ . It can occur even in high-quality superconductor samples whose bulk properties are described by a free energy functional with spatially constant coefficients. The surface pair breaking is usually considered as a specific result of self-consistent calculations, which should be taken into account for a quantitative description of relevant problems. However, it is also able to modify a number of physical processes qualitatively. Thus, in unconventional superconductors the surface pair break-

ing is sensitive, under certain conditions, to the crystal to surface orientation, and disregarding its effect can result in qualitative changes of orientation dependence of the Josephson current characteristics, including a region of crystal orientations, where the  $0 - \pi$  transition takes place with a change of temperature.<sup>30,31</sup> The surface pair breaking can result in zero-energy surface Andreev bound states<sup>32</sup>, which, for instance, modify anomalously the low temperature magnetic response of superconductors.<sup>33-37</sup> An inhomogeneous pair amplitude can occur in the near-surface region both in the presence and in the absence of the Andreev states, and its direct influence on  $\lambda$  is of interest. For now only a little is known about the effect of pair-amplitude inhomogeneity on the penetration depth. Therefore, obtaining a corresponding solution, even within the simplest framework, would be desirable to facilitate analysis of the problem.

This paper addresses  $\lambda$  within the London and the GL theories, assuming the local properties of a massive superconductor to have a smooth dependence on the distance from its plane boundary over a characteristic scale  $\ell$ . The theory developed allows one to express the global penetration depth  $\lambda$  via a spatially dependent local one  $\lambda_{loc}$ , without using a particular spatial dependence of  $\lambda_{loc}$ . Since  $\lambda_{loc}$  and the associated superfluid density are local quantities, while the magnetic penetration depth  $\lambda$  is a spatially independent (global) quantity, they cannot be related to each other locally. Therefore, obtaining nonlocal relationships between  $\lambda$  and  $\lambda_{loc}$ , which directly links  $\lambda$  with local superconductor characteristics, should be of interest.

Changes of  $\lambda$  will be found below both for the small-scale  $\ell \ll \lambda$  and the large-scale  $\ell \gg \lambda$  inhomogeneities in the near-surface region. In the presence of large-scale inhomogeneities  $\ell \gg \lambda$ , the near-surface superfluid density controls the superconducting screening. If the superfluid density is either suppressed, or enhanced on the surface as compared to its bulk value, it weakens or reinforces the screening. On the face of it, the modified  $\lambda$  is obtained from the conventional one by substituting, in the zeroth order in  $\lambda/\ell$ , the surface value of the superfluid density for its bulk value. However, this proves to be correct only under certain conditions and is no longer valid

with the superconductivity strongly suppressed close to the surface. The complete suppression on the surface would result in vanishing screening, which is incompatible with the presence of the superconductivity in the bulk. Generic characteristic values of  $\lambda$  pertaining to this case are identified in the paper. A parametric crossover of the two screening regimes discussed here is described taking into account a spatial dependence of the local penetration depth. In particular, the penetration depth of strongly type-I superconductors in the presence of a pronounced surface pair breaking is obtained near  $T_c$ .

The screening usually shows a weak sensitivity to the small-scale inhomogeneities and to the surface pair breaking or pair formation in strongly type-II superconductors (where  $\lambda \gg \xi \sim \ell$ ). Here the magnetic field mostly varies over distances  $\gg \ell$ , where the superconductor is practically uniform. This feature underlies the London theory, allowing one to disregard the influence of small-scale inhomogeneities on the Meissner effect in numerous strongly type-II superconductors, both conventional and unconventional. However, a few reasons, which restrict the applicability of the arguments presented, make the effects of the small-scale inhomogeneities on  $\lambda$ , including those beyond the London limit, of real interest and importance. The corresponding first-order correction  $\lambda^{(1)}/\lambda \sim \ell/\lambda \ll 1$  can amount to about 10%, being well within the resolution of the present-day experiments, which can identify small changes of  $\lambda$  up to  $\Delta\lambda/\lambda \sim 0.5\%$  in high-temperature superconductors.<sup>2,3,36,38,39</sup> The surface pair breaking in strongly type-II superconductors results in a similar term  $\lambda^{(1)}/\lambda \sim \kappa^{-1}$ , where  $\kappa$  is the GL parameter. Estimating  $\kappa$  for high-temperature superconductors as  $\lesssim 100$ , one concludes that even in such a case the correction could be resolved. Therefore, a quantitative description of  $\lambda^{(1)}$  is required. In the present paper its general analytical form is obtained within the framework outlined above.

## II. SMALL-SCALE INHOMOGENEITIES

The supercurrent in isotropic superconductors can be written within the GL theory as

$$\mathbf{j} = -\frac{cf^2}{4\pi\lambda_L^2} \left( \frac{\Phi_0}{2\pi} \nabla\chi + \mathbf{A} \right). \quad (1)$$

Here  $\Phi_0 = \pi\hbar c/|e|$  is the superconductor flux quantum and the normalized modulus of the order parameter  $f$  is equal to unity in the bulk. Not only the current density  $\mathbf{j}$ , the vector potential  $\mathbf{A}$  and the order-parameter phase  $\chi$ , but also  $f$  and the local London penetration depth  $\lambda_L$  can in general depend on spatial coordinates.

Let the magnetic field be applied along the  $z$  axis to a massive isotropic superconductor ( $x > 0$ ) with a plane boundary at  $x = 0$ . Inhomogeneities of the material and of the order parameter are assumed to appear, for either physical or technological reasons, solely

due to the presence of the boundary. Specifically, the field  $\mathbf{h}(x) = h(x)\mathbf{e}_z$  and the local penetration depth  $\lambda_{\text{loc}}(x) = \lambda_L(x)/f(x)$  are considered to depend only on the distance  $x$  from the surface. Then the screening supercurrent density  $\mathbf{j}(x) = j(x)\mathbf{e}_y$  flows along the  $y$  axis. Far inside the sample, at  $x \gg \ell$ , the superconductor is assumed to be homogeneous with constant bulk values  $\lambda_{Lb}$  of the local London penetration depth and  $f_b = 1$  of the normalized modulus. Also, the applied field is considered to be substantially less, than the critical fields, and to produce a negligibly small influence on  $\lambda_{\text{loc}}(x)$ .

Taking the gauge  $\text{div } \mathbf{A} = 0$  for the problem in question, one can choose the order-parameter phase to be spatially constant, which results in the London relation  $\mathbf{j}(x) = -c\mathbf{A}(x)/4\pi\lambda_{\text{loc}}^2(x)$ , and eventually in the one-dimensional equation for the Meissner effect in a linear approximation in the magnetic field:

$$A''(x) - \lambda_{\text{loc}}^{-2}(x)A(x) = 0. \quad (2)$$

If the characteristic scale  $\ell$  of a spatial dependence of  $\lambda_{\text{loc}}^{-2}(x)$  satisfies the strong inequality  $\ell \ll \lambda_{Lb}$ , then  $h(x)$  mostly varies in the space region, where  $\lambda_{\text{loc}}^{-2}(x)$  is nearly constant and equal to  $\lambda_{Lb}^{-2}$ . The scale can originate from the sample inhomogeneities near the surface, and/or from the profile  $f(x)$  induced by the surface pair breaking in strongly type-II superconductors. The corresponding magnetic penetration depth  $\lambda^{(0)}$ , taken in the zeroth approximation in  $\ell/\lambda_{Lb}$ , coincides with  $\lambda_{Lb}$ . This standard result relates  $\lambda$  to the superfluid density in the bulk and makes it entirely independent of the small-scale inhomogeneities and of the underlying boundary conditions for the order parameter.

The correction  $\lambda^{(1)}$  to the penetration depth of the first order in  $\ell/\lambda_{Lb}$  will be obtained here without resorting to a solution of (2) for any particular spatial dependence of  $\lambda_{\text{loc}}(x)$ . Multiplying (2) by  $h(x)$  and integrating all the terms over the superconducting region, one gets

$$h_0^2 - \lambda_{\text{loc}}^{-2}(0)A_0^2 = \int_0^\infty \frac{d\lambda_{\text{loc}}^{-2}(x)}{dx} A^2(x) dx. \quad (3)$$

As  $\lambda \gg \ell$ ,  $A(x)$  varies only a little over  $\ell$ , while  $d\lambda_{\text{loc}}^{-2}(x)/dx$  almost vanishes at  $x \gg \ell$ . Hence, one can expand  $A^2(x)$  in (3) in powers of  $x$  and, as an approximation, keep only the first two terms. Then the standard definition of the penetration depth  $\int_0^\infty h(x) dx = \lambda h_0$ , the equality  $h = dA/dx$  and the relation  $A_0 = -\lambda h_0$  between the surface values of  $h(x)$  and  $A(x)$ , lead to  $A^2(x) \approx (\lambda^2 - 2\lambda x) h_0^2$  in the near-surface region. With this expression, Eq. (3) is reduced to a quadratic equation for  $\lambda$  resulting in the following solution

$$\lambda \approx \lambda^{(0)} + \lambda^{(1)} = \lambda_{Lb} \left( 1 + \lambda_{Lb} \int_0^\infty x \frac{d\lambda_{\text{loc}}^{-2}(x)}{dx} dx \right). \quad (4)$$

Eq. (4) relates the global  $\lambda$  and the local  $\lambda_{\text{loc}}(x)$  penetration depths to each other, taking into account the surface contribution to  $\lambda$  within the first order in  $\ell/\lambda_{Lb}$ .

Consider, for example, the particular spatial profile  $\lambda_{\text{loc}}^{-2}(x) = (1 - e^{-x/\ell}) \lambda_{Lb}^{-2}$ . Here  $\lambda_{\text{loc}}^{-2}(x)$  vanishes at  $x = 0$  and approaches the bulk value  $\lambda_{Lb}^{-2}$  with increasing distances  $x \gtrsim \ell$ . Substituting the dependence  $\lambda_{\text{loc}}^{-2}(x)$  in (4), one obtains  $\lambda = \lambda_{Lb} + \ell$ . This simple result clearly agrees with a strong suppression of the screening of the magnetic field on the scale  $\ell$  near the surface and with a subsequent screening over the scale  $\lambda_{Lb}$ .

The quantitative character of (4) is revealed once the spatial profile of  $\lambda_{\text{loc}}^{-2}(x)$  is identified unambiguously, with no further assumptions made about its specific form, as done in the preceding paragraph. In section III quantitative analysis is used to describe the proximity effect on the penetration depth.

### III. PROXIMITY TO THE SURFACE IN STRONGLY TYPE-II SUPERCONDUCTORS

Proximity to the plane interface generally induces a one-dimensional spatial dependence of the pair amplitude  $f(x)$  and, hence, of the quantity  $\lambda_{\text{loc}}^{-2}(x) = f^2(x) \lambda_L^{-2}$  in massive homogeneous superconducting samples. The possibility of a broken translational symmetry is disregarded here, since it has been established theoretically at quite low temperatures and only in thin superconducting films.<sup>40</sup> Therefore, the effect of the surface pair breaking or pair formation on  $\lambda$  in strongly type-II superconductors can be described within the GL theory, based on (4), without any model assumptions.

To obtain the corresponding  $\lambda$ , the bulk and the surface contributions to the GL free energy are considered for the  $s$ -wave or  $d_{x^2-y^2}$ -wave homogeneous superconductors in the absence of the magnetic field:

$$\mathcal{F} = \int_V \left( K |\nabla \Psi|^2 + a |\Psi|^2 + \frac{b}{2} |\Psi|^4 \right) dV + \int_S g |\Psi|^2 dS. \quad (5)$$

Spatially dependent solutions of the corresponding GL equations with  $f_\infty = 1$  are well known (see, e.g.,<sup>11–13</sup>). They take the form

$$f(x) = \tanh \left( \frac{x + x_0}{\sqrt{2}\xi} \right), \quad f(x) = \coth \left( \frac{x + x_0}{\sqrt{2}\xi} \right) \quad (6)$$

for the pair breaking and the pair forming surfaces respectively. A relationship between  $x_0$  and the original coefficients in (5) will be established below and used for describing  $\lambda$ .

The surface term in (5) results in the surface pair breaking ( $g > 0$ ) or pair formation ( $g < 0$ ). The dimensionless parameter  $g_\delta = g\xi/K$ , containing the coherence length of the GL theory  $\xi = \sqrt{K/|a|}$ , characterizes the strength of the surface effect and, in particular, determines the surface value  $f_0$  for the order parameter modulus  $|\Psi(x)| = (|a|/b)^{1/2} f(x)^{29}$ :

$$f_0 = \frac{1}{\sqrt{2}} \left( \sqrt{2 + g_\delta^2} - g_\delta \right). \quad (7)$$

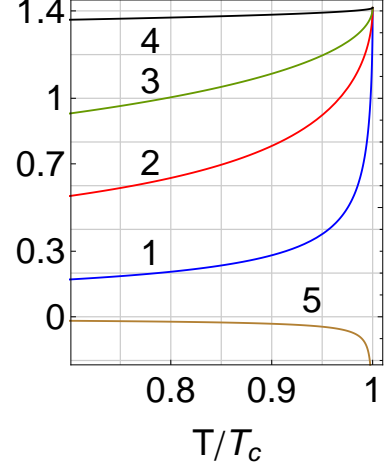


FIG. 1.  $\lambda^{(1)}/\xi$  as a function of the temperature taken for strongly type-II superconductors with various  $g_{\delta 0}$ : (1)  $g_{\delta 0} = 0.1$  (2)  $g_{\delta 0} = 0.4$  (3)  $g_{\delta 0} = 1$  (4)  $g_{\delta 0} = 10$  (5)  $g_{\delta 0} = -0.01$ .

Combining (7) and (6) results in an analytical relationship between  $x_0$  and  $g_\delta$ :

$$x_0 = \frac{\xi}{\sqrt{2}} \ln \frac{\sqrt{2} + \sqrt{2 + g_\delta^2} - g_\delta}{|\sqrt{2} - \sqrt{2 + g_\delta^2} + g_\delta|}. \quad (8)$$

In the case of  $g_\delta = 0$ , when  $x_0 \rightarrow \infty$ ,  $f_0 = 1$  the pair activity of the surface vanishes and  $\lambda$  coincides with  $\lambda_L = \hbar c b^{1/2} / 4 \sqrt{2\pi} |e| K^{1/2} |a|^{1/2}$ . The value  $x_0 = 0$  ( $g_\delta \rightarrow \infty$ ) brings about complete suppression of the order parameter on the surface  $f_0 = 0$ .

The quantities  $g_\delta$  and  $x_0$ , which control the surface value of the order parameter in (7) and (6), can be determined both experimentally and theoretically, and therefore taken as known in considering the penetration depth. The microscopic theory for coefficients of the bulk free energy in (5) has been identified for isotropic  $s$ -wave<sup>41,42</sup> and more recently for  $d$ -wave<sup>43,44</sup> superconductors. As for the coefficient  $g$  in (5) and/or the corresponding order-parameter suppression, they have been also studied microscopically for various cases.<sup>11–14,17–23</sup> Experimentally, the surface values of the order parameter can be established using a scanning tunneling microscopy method with a superconducting tip<sup>45</sup>.

On account of (8), the integration in (4) with each of the functions in (6) results in the following contribution from the surface pair breaking, or pair formation, to  $\lambda$ :

$$\lambda^{(1)} = \sqrt{2}(1 - f_0)\xi = \sqrt{2} \left[ 1 - \frac{1}{\sqrt{2}} \left( \sqrt{2 + g_\delta^2} - g_\delta \right) \right] \xi. \quad (9)$$

Eq. (9) holds provided  $|\lambda^{(1)}| \ll \lambda_L$ , and irrespective of the sign of  $g_\delta$ .

The first correction (9) to  $\lambda$  depends linearly on the surface value of the order parameter  $f_0$ . The temperature dependence of  $\lambda^{(1)}$  is controlled in (9) by  $\xi = \kappa^{-1} \lambda_L$

both directly and via the quantity  $g_\delta$ . Within the GL theory  $\xi = \xi_0 \tau^{-1/2}$  and  $g_\delta = g_{\delta 0} \tau^{-1/2} \equiv g_{\delta 0} K^{-1} \tau^{-1/2}$ , where  $\tau = 1 - (T/T_c)$ . The temperature dependence of  $\lambda^{(1)}/\xi$  is shown in Fig. 1 for various  $g_{\delta 0}$ . For a weak pair breaking  $|g_\delta| \ll 1$  the correction is the product of two small parameters  $\lambda^{(1)}/\lambda_L = g_\delta \kappa^{-1}$ , and  $|\lambda^{(1)}| = |g_\delta| \xi \ll \xi$ . However, for a strong surface pair breaking  $g_\delta \gtrsim 1$  the correction can exceed  $\xi$ . If the order parameter is completely suppressed on the boundary ( $g_\delta \rightarrow \infty$ ), the answer is  $\lambda \approx [1 + (\sqrt{2}/\kappa)] \lambda_L$ . In this limit the relative correction  $\lambda^{(1)}/\lambda_L$  is independent of the temperature:  $\lambda^{(1)} = (\sqrt{2}/\kappa) \lambda_L = \sqrt{2} \xi$ .

Unlike the pair breaking surfaces, a strongly pair forming boundary with  $g_\delta \ll -1$  generates a small characteristic scale and, as a result, the applicability of the GL theory to such systems is generally restricted. When  $-g_\delta \gg 1$ , and the superconductivity is significantly enhanced near the boundary, one obtains  $\Delta\lambda = -2|g_\delta|\xi$  from (9). Here the condition  $|\Delta\lambda^{(1)}| \ll \lambda_L$  needed for the applicability of (9), results in an additional restriction  $2|g_\delta| \ll \kappa$ .

The simple approach, used here for deriving (4) and (9) for  $\lambda^{(1)}$ , does not directly apply to obtaining higher order terms, since the derivatives of  $\lambda_{\text{loc}}^{-2}(x)$  enter the expressions for higher spatial derivatives of  $h(x)$ .

#### IV. LARGE-SCALE INHOMOGENEITIES

Under the condition  $\ell \gg \lambda$ , the coefficient  $\lambda_{\text{loc}}^{-2}(x)$  in Eq. (2) can be considered as a slow function of  $x$ . In the simplest case and in the zeroth approximation in  $\lambda/\ell$ , one can take  $\lambda \approx \lambda_{\text{loc}}(0) = \lambda_L(0)/f(0)$ . In particular, one has  $\lambda \gg \lambda_{Lb}$  as a result of a pronounced surface pair breaking  $f(0) \ll 1$  in strongly type-I superconductors. However, this simple formula for  $\lambda$  is generally incorrect, as it predicts no screening when  $\lambda_{\text{loc}}^{-2}(x)$  vanishes on the surface locally. Since the screening does not actually vanish due to the presence of superconductivity in the sample, one should keep in (2) all the relevant terms in the expansion of  $\lambda_{\text{loc}}^{-2}(x)$  in the near-surface region  $x \lesssim \lambda \ll \ell$ .

For the spatial dependence of  $\lambda_{\text{loc}}^{-2}(x)$ , its standard Taylor expansion at  $x = 0$  will be put to use here. If the first spatial derivative of  $\lambda_{\text{loc}}^{-2}(x)$  is not anomalously small at  $x = 0$ , then the constant and linear in  $x$  terms should be kept in the expansion. Thus Eq. (2) transforms to the Airy equation, and the solution, which vanishes far inside the superconductor, leads both to the spatially dependent vector potential and the magnetic field. This results eventually in the following penetration depth  $\lambda = -A(0)/h(0)$ :

$$\lambda = \frac{K_{1/3}(2\beta/3)}{K_{2/3}(2\beta/3)} \lambda_{\text{loc}}(0), \quad (10)$$

where

$$\beta = \lambda_{\text{loc}}^{-3}(0) \left( \frac{d\lambda_{\text{loc}}^{-2}(x)}{dx} \right)_0^{-1}. \quad (11)$$

The asymptotic expressions for the Macdonald functions, which do not depend on their order<sup>46</sup>, can be used in (10) provided  $\beta \gg 1$ . This determines the domain of applicability of the simple result  $\lambda = \lambda_{\text{loc}}(0)$ . The opposite limit  $\beta \ll 1$  is brought about by the anomalously small (or vanishing) value of  $\lambda_{\text{loc}}^{-3}(0)$  as compared to  $(d\lambda_{\text{loc}}^{-2}(x)/dx)_0$ . In this case one substitutes in (10)  $K_\nu(z) \approx \Gamma(\nu) 2^{\nu-1} z^{-\nu}$ <sup>46</sup> and obtains

$$\lambda \approx 1.3717 \left( \frac{d\lambda_{\text{loc}}^{-2}(x)}{dx} \right)_0^{-1/3}. \quad (12)$$

The last result identifies a characteristic scale  $\lambda \sim (\lambda_{Lb}^2 \ell)^{1/3}$ , if the standard qualitative estimate used does work. This leads to  $\lambda \gg \lambda_{Lb}$  under the condition  $\ell^{1/3} \gg \lambda_{Lb}^{1/3}$ , while for the strong inequality  $\lambda \ll \ell$  to be valid, the relation  $\ell^{2/3} \gg \lambda_{Lb}^{2/3}$  has to hold. This is in keeping with the large-scale character of the inhomogeneities considered.

The linear spatial behavior of  $\lambda_{\text{loc}}^{-2}(x)$  in the near-surface region and its complete suppression on the surface entail the square-root dependence of  $\lambda_{\text{loc}}^{-1}(x) \propto \sqrt{x}$  as well as its infinite steepness  $d\lambda_{\text{loc}}^{-1}(x)/dx \propto x^{-1/2}$  at  $x = 0$ . Previous qualitative analysis of this particular case<sup>7</sup> well agrees with the quantitative result (10) obtained here. However, the expression (10) is insufficient for studying the problem. It completely ignores the possibility of the regular Taylor expansion of  $\lambda_{\text{loc}}^{-1}(x)$  at  $x = 0$ , where vanishing  $\lambda_{\text{loc}}^{-1}(0)$  is compatible with a finite steepness  $(d\lambda_{\text{loc}}^{-1}(x)/dx)_0$ . In the latter case, the quantities  $\lambda_{\text{loc}}^{-2}(x)$  and  $d\lambda_{\text{loc}}^{-2}(x)/dx = 2\lambda_{\text{loc}}^{-1}(x) (d\lambda_{\text{loc}}^{-1}(x)/dx)$  can only vanish simultaneously. If they both vanish at  $x = 0$ , (10) and its limiting form (12) diverge, and the above estimate fails. As a consequence, to obtain the correct result from (2) one should generally keep the first three terms, including the quadratic one in  $x$ , in the expansion of  $\lambda_{\text{loc}}^{-2}(x)$ . The effect of surface/interface pair breaking on  $\lambda$  is an example of this type. As seen from (6) in the case  $x_0 = 0$  ( $f_0 = 0$ ), the GL theory predicts the linear (and not a square root) behavior  $\lambda_{\text{loc}}^{-1}(x) \approx x/\sqrt{2}\xi\lambda_L$  and, correspondingly, the quadratic behavior of  $\lambda_{\text{loc}}^{-2}(x)$  in the near-surface region.

With the quadratic form on  $x$  taken for  $\lambda_{\text{loc}}^{-2}(x)$ , Eq. (2) can be transformed to that of the confluent hypergeometric functions. One can significantly simplify the solution by specifying the condition of the anomalous smallness of  $\lambda_{\text{loc}}^{-1}(0)$  as

$$\alpha \equiv \lambda_{\text{loc}}^{-1}(0) \left| \frac{d^2 \lambda_{\text{loc}}^{-1}(x)}{dx^2} \right|_0 \left( \frac{d\lambda_{\text{loc}}^{-1}(x)}{dx} \right)_0^{-2} \ll 1 \quad (13)$$

and making an additional assumption  $\alpha\beta \ll 1$ .



Under these conditions the penetration depth is

$$\lambda = \frac{K_{1/4}(\beta)}{K_{3/4}(\beta)} \lambda_{\text{loc}}(0), \quad (14)$$

where both limiting cases  $\beta \gg 1$  and  $\beta \ll 1$  as well as their crossover are still allowed within the framework  $\alpha\beta \ll 1$  and  $\alpha \ll 1$ .

For  $\beta \gg 1$  both formulas (10) and (14) lead to the equality  $\lambda = \lambda_{\text{loc}}(0)$ . However, in the limit  $\beta \ll 1$  one gets from (14)

$$\lambda \approx 1.4793 \left( \frac{d\lambda_{\text{loc}}^{-1}(x)}{dx} \right)_0^{-1/2}. \quad (15)$$

The standard qualitative estimate now results in the characteristic scale  $\lambda \sim \sqrt{\lambda_{Lb}\ell}$ , which differs from the one following from the Airy equation in the similar limit  $\beta \ll 1$ . The conditions  $\lambda_{Lb} \ll \lambda \ll \ell$  are satisfied here provided  $\ell^{1/2} \gg \lambda_{Lb}^{1/2}$ .

## V. PROXIMITY TO THE SURFACE IN STRONGLY TYPE-I SUPERCONDUCTORS

Eqs. (10)-(15) obtained above can be applied to any superconductor, where a local magnetic response, characterized by  $\lambda_{\text{loc}}$ , manifests a one dimensional spatial dependence  $\lambda_{\text{loc}}(x)$  under the condition  $\ell \gg \lambda$ . Let us consider, based on Eqs. (14) and (15), the penetration depth  $\lambda$  influenced by the surface pair breaking in strongly type-I superconductors, where  $\ell \sim \xi \gg \lambda_{Lb}$ . Such a consideration is justified within the GL theory, which is restricted here by temperatures near  $T_c$  also due to a nonlocal character of the magnetic response of the type-I superconductors at lower temperatures.

If a spatial dependence of  $\lambda_{\text{loc}}^{-1}(x)$  is controlled entirely by the surface pair breaking, explicit expressions for various spatial derivatives of  $\lambda_{\text{loc}}^{-1}(x)$  can be found based on (6). In particular, one obtains from (11), (13) and (6) under the condition  $g_\delta^2 \gg 1$

$$\alpha = \frac{1}{g_\delta} \left( \sqrt{g_\delta^2 + 2} - g_\delta \right) \approx g_\delta^{-2} \ll 1, \quad (16)$$

$$\beta = \frac{\sqrt{g_\delta^2 + 2} - g_\delta}{2^{3/2} \kappa g_\delta} \approx 2^{-3/2} \kappa^{-1} g_\delta^{-2}. \quad (17)$$

The condition  $\alpha\beta \sim \kappa^{-1} g_\delta^{-4} \ll 1$ , presumed in (14) for  $\kappa \ll 1$  and  $g_\delta^2 \gg 1$  considered, does involve both limiting cases  $\kappa \ll g_\delta^{-2}$  and  $g_\delta^{-2} \ll \kappa \ll 1$ , and their crossover. In the former case  $\lambda \approx \lambda_{\text{loc}}(0) \approx \sqrt{2} g_\delta \lambda_{Lb}$ . For  $g_\delta^{-2} \ll \kappa$  the penetration depth is

$$\lambda \approx 1.7592 \sqrt{\xi \lambda_{Lb}}. \quad (18)$$

It represents a characteristic scale  $\sim \sqrt{\xi \lambda_{Lb}}$ , which can substantially exceed  $\lambda_{Lb}$  retaining much less than  $\xi$ . The behavior of  $\lambda/\sqrt{\xi \lambda_{Lb}}$ , obtained with the solution containing the confluent hypergeometric functions, is shown

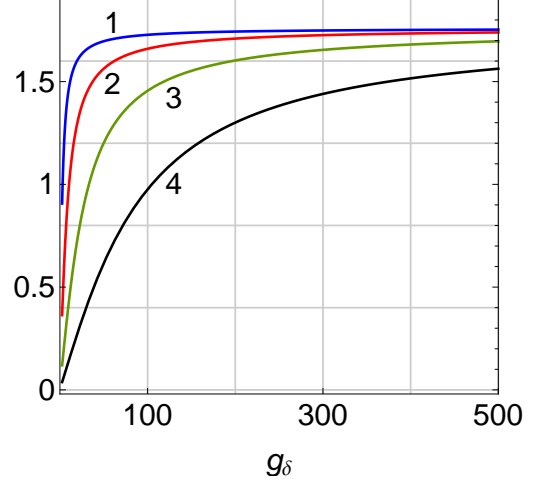


FIG. 2.  $\lambda/\sqrt{\xi \lambda_{Lb}}$  as a function of  $g_\delta$  taken for strongly type-I superconductors with various  $\kappa$ : (1)  $\kappa = 0.1$  (2)  $\kappa = 0.01$  (3)  $\kappa = 0.001$  (4)  $\kappa = 0.0001$ .

in Fig. 2 in a wide range of  $g_\delta$  for various  $\kappa \ll 1$ . All the curves approach their common asymptotic value 1.7592 at  $g_\delta^2 \gg \kappa^{-1}$ .

## VI. CONCLUSIONS

This paper has theoretically studied the effect of near-surface inhomogeneities on the magnetic penetration depth under a weak applied magnetic field. When inhomogeneities of the order parameter are induced in a homogeneous sample due to the proximity to the surface, the penetration depth is obtained as a function of the surface pair breaking parameter, within the GL theory with no model assumptions, for both strongly type-II and strongly type-I superconductors. Since all the coefficients of the GL free energy functional, including the coefficient  $g$  of the surface term, can be specified based on the measurements, which are independent of the study of  $\lambda$ , they can be taken as known in considering the penetration depth.

The theory developed allows one to express the global penetration depth  $\lambda$  via a spatially dependent local one  $\lambda_{\text{loc}}(x)$ , without resorting to a particular spatial dependence of  $\lambda_{\text{loc}}(x)$ . The changes of  $\lambda$  are found for both the small-scale and the large-scale inhomogeneities. In the latter case the characteristic lengths of the superconducting screening are shown to differ significantly from  $\lambda_{Lb}$ , when the superconductivity is strongly suppressed on the surface. Changes of  $\lambda$  due to the small-scale inhomogeneities are obtained and shown to be, as a rule, well within the present experimental resolution.

The results obtained apply to the samples with one dimensional inhomogeneous profiles of all the quantities, as is the case in certain conditions. Similar problems of two

or three dimensional character are of great interest and, in general, substantially more complicated. Only in some specific cases can they incorporate the one dimensional problem in question as their ingredient. For example, modern experiments can identify the temperature dependence of both the global penetration depth<sup>3,36,39</sup> and the local one with respect to lateral coordinates along the surface.<sup>38</sup> Due to the presence of sample inhomogeneities, it is not an easy task to compare such results.<sup>47</sup> In the simplest case of different large-scale surface regions, the

global penetration depth can be approximated as the average of the one dimensional results for  $\lambda$  over the lateral coordinates.

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